Tutorial 1

1. a) Show that
\[ E^2 = \frac{\beta}{Z} \frac{\partial^2 Z}{\partial \beta^2} \]

b) Show that the standard deviation in the value of energy is given by
\[ \sigma = \sqrt{E^2 - \left( \langle E \rangle \right)^2} = \sqrt{\frac{\partial^2}{\partial \beta^2} \ln Z} \]

2. Consider a one level system having energy
\[ \varepsilon = -N k_B T \ln \left( \frac{V}{V_0} \right) \]
Where \( V_0 \) is a constant.

   a) Write down the partition function for this system
   b) Calculate the average pressure for this system as a function of volume and temperature.

3. Calculate the mean energy for a diatomic gas having \( N \) molecules.

4. Consider a system having two non-degenerate microstates, with energy \( \varepsilon_1 \) \( \varepsilon_2 \). The probability of occupancy of these states is given by \( P_1 \) and \( P_2 \), and a characteristic temperature \( \theta \) is defined as:
\[ \theta = \frac{\varepsilon_2 - \varepsilon_1}{k_B} = \frac{\varepsilon_g}{k_B} \]

   a) Find \( P_2 \), when \( T = 0.01 \theta \) and \( 0.1 \theta \).
   b) Prove that the ensemble average energy of the two-level system is given by :
\[ \langle E \rangle = \frac{\varepsilon_1 + \varepsilon_2 \exp \left( -\frac{\theta}{T} \right)}{1 + \exp \left( -\frac{\theta}{T} \right)} \]
Hence show that for \( \theta = T \), \( E = 0.731 \varepsilon_1 + 0.269 \varepsilon_2 \).

5. The partition function of a system is given by \( \ln Z = \alpha T^4 V \) where \( \alpha \) is a constant, \( T \) is the absolute temperature and \( V \) is the volume. Calculate the internal energy, the pressure and the entropy.