THE MASS OF THE ELECTRON IN GaAs/AlGaAs BY
SHUBNIKOV-DE HAAS EFFECT AND
THE SPIN-CHARGE LOCKING

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ABSTRACT

The energy of the free electron gas is calculated in three dimensions. In this calculation an integral over the wave vector space occurs which was solved by Dingle. In the low temperature approximation the integral over the Fermi distribution leads to $x/\sinh x$ type expression, called the Dingle’s formula. The spin symmetry is found to modify this formula which determines the oscillation amplitude of resistivity as a function of magnetic field, called the Shubnikov-de Haas effect. The theory introduces the effective charge so that the cyclotron frequency gets fractionalized resulting into $m/\nu\pm$ which for $\nu\pm=1$ becomes $m$, the electron mass. Thus we have taken into account, the spin-charge fractions, to obtain the correct mass. For example, at certain magnetic field $1.5m$ is found instead of $m$. The Shubnikov-de Haas effect uses quantization of Landau levels but not the flux quantization. Hence we find that there is a “quantized Shubnikov-de Haas effect” which measures $m/h^2$. We determine that when fractional values of $\nu\pm$ are taken into account, the mass of the electron, equal to the band mass in GaAs/AlGaAs is obtained.

INTRODUCTION

The electrons in two dimensions, in a magnetic field give harmonic oscillator type energy levels with cyclotron frequency, $\omega_c=\epsilon B/mc$. The electron has only two levels corresponding to spin $\pm 1/2$ so that the energy level difference is $g\mu_B B=h\omega_c$. Since for free electrons, the orbital angular momentum is zero, $g=2$ the factor $g/2$ being equal to 1 completely disappears. The energy levels then become equally spaced, $h\omega_c(n+1/2)$. When magnetic field is varied, different values of $n$ cross the Fermi energy at different fields resulting into oscillations in the resistivity as a function of magnetic field which is called the Shubnikov-de Haas effect. The oscillations due to the same effect observed through the susceptibility are called the de Haas-van Alphen effect. The physical origin of the Shubnikov-de Haas effect is thus the same as that of the de Haas-van Alphen
effect. The period of oscillations can be used to determine the mass of the electron, \( m_e \), when \( e \) and \( c \) are known and \( B \) is measured. In a solid this mass equals that deduced from the band structure. Usually, the harmonic oscillator type of levels do not split and it is difficult to fractionalize them. In the Dirac equation \( L+S \) is conserved but \( L-S \) is not. The \( L\pm S \) can be conserved but then the resonance for both signs need not occur at the same field because \( g/2 \) ceases to be 1 and we have to introduce two values instead of one, \( g_\pm/2=\nu_\pm \) for the two signs. It is possible to absorb the factor in the change of the electron so that the charge becomes \( e\nu_\pm \) instead of \( e \). If this factor in the charge is ignored, it will appear in the mass as \( \nu_\pm eB/mc=eB/[(m/\nu_\pm)c] \) so that the measurement of the mass of the electron is affected. Therefore, it is important to take into account the spin-charge relationship found in ref.[1,2]. The experimental value of \( g \) is often shifted from the present value due to many-body effects. There may be three distinct \( g \) values in the picture, \( g \) and two values of \( g_\pm \).

In the present paper, we report the mass of the electron corrected for the spin-charge effect which also uses the negative spin. We introduce the flux quantization in the Shubnikov-de Haas effect to find a new quantum effect, which we call as the “quantized Shubnikov-de Haas effect” using which \( m/h^2 \) can be measured.

**THEORY**

The electrons in two dimensions behave like a harmonic oscillator with a single cyclotron frequency. It is possible to produce harmonics with integer multiple of the cyclotron frequency but it is usually very difficult to split the frequency of the harmonic oscillator. In a magnetic field the cyclotron frequency becomes equal to the Zeeman energy. One of the recent theories of the cyclotron frequency is given by Vavilov[3]. In this theory only a single frequency, \( h\omega_c \), has been considered. The harmonic oscillator type eigen values are found for the electrons in a magnetic field with the transition frequency equal to that of the cyclotron frequency. Usually, an integral over the Fermi distribution arises which can not be solved analytically for finite limits. However, at low temperatures, where de Haas-van Alphen effect is observed, Dingle[4] used infinite limits to obtain the value, \( \pi\delta/\sinh(\pi\delta) \) where \( \delta=2\pi k_B T/\mu_B B \). This is called Dingle’s formula and it is valid for a single cyclotron frequency, which does not permit fractional charges. In order to allow fractional charges, as they are observed in the quantum Hall effect, we may consider them also in the Shubnikov-de Haas effect. Therefore, the theory with a single cyclotron frequency breaks down. We consider the spin-charge effect for the introduction of multiple solutions of the cyclotron frequency. The higher multiples of the frequency arise, so that it becomes \( n h\omega_c, \) and
fractional frequencies also occur. Therefore, we introduce the orbital as well as the spin quantum numbers so that the frequency is given by $\nu_{\pm} \hbar \omega_c$ where,

$$\nu_{\pm} = \frac{l + (1/2) \pm s}{2l + 1}.$$  \hspace{1cm} (1)

In the Shubnikov-de Haas oscillations, it is sufficient to consider only the phase factors so that the oscillating resistivity can be written as,

$$\delta \rho_{xx}(\varepsilon) = \rho_0 \Sigma_p \gamma_{th} c_{\alpha,\beta} \exp(- \frac{p\pi}{\nu_{\pm} \omega_c \tau_q}) \cos[2\pi p \left( \frac{\varepsilon}{\nu_{\pm} \hbar \omega_c} - \frac{1}{2} \right)]$$  \hspace{1cm} (2)

where,

$$\gamma_{th} = \frac{2\pi^2 pk_y T}{\sinh[2\pi^2 pk_y T/(\hbar \omega_c \nu_{\pm})]}$$  \hspace{1cm} (3)

so that one value is obtained for $+$ sign and another for $-$ sign. The constant resistivity $\rho_0$ is the mean value around which oscillations occur, $p$ is an integer and $c_{\alpha,\beta} = 16/3 \pi \alpha$ with $\alpha = 2\pi \omega_c \tau_q$. The amplitude of these oscillations gives $m/\nu_{\pm}$ rather than $m$, the mass of the electron. For $l = 0$, spin $1/2$ and $\nu_{\pm} = 1$, the measured mass is equal to the band mass, $m_{\text{band}}$. For $l = 1$, spin $+1/2$, $\nu_{\pm} = 2/3$ so that the cyclotron frequency becomes $(2/3) \hbar \omega_c$ and the mass of the electron becomes $(3/2)m$. In this way, a mass of $m/m_{\text{band}} = 1.5$ is predicted from which the mass is determined by taking into account the multiplying factor.

The cos factor in the $\delta \rho_{xx}$ is zero when,

$$2\pi p \left( \frac{\varepsilon}{\nu_{\pm} \hbar \omega_c} - \frac{1}{2} \right) = \frac{\pi}{2}$$  \hspace{1cm} (4)

so that the resistivity vanishes when,

$$B_0 = \frac{2mc\varepsilon}{\nu_{\pm} e\pi(n_1 + 2p)}.$$  \hspace{1cm} (5)

For $n_1 = 1, p = 1, n_1 + 2p = 3$,

$$B_0 = \frac{2mc\varepsilon}{3\nu_{\pm} e\pi}.$$  \hspace{1cm} (6)

At these fields, the resistivity is zero. Since there are two values of $\nu_{\pm}$ such as $1/3$ and $2/3$, there is a symmetry exhibited by field values given by $B_0$ when the values of $\nu_{\pm}$ are substituted.

**ZERO RESISTANCE STATE**
The zero-resistance state found upon shining the Shubnikov-de Haas oscillations with microwaves has been observed in several experiments[5-7]. A case, in which the excited state life time is zero has been discussed by Lee and Leinaas[8]. An effort is made by Durst et al[9] to obtain the theory of zero-resistance state but the fractions do not emerge correctly in this theory. In case, the mass of the electron is obtained, it is necessary to include the factors, $\nu_\pm$. The quantity measured is $m/\nu_\pm$ so that when $\nu_\pm=2/3$ a factor of $3/2$ appears and the mass becomes $(3/2)m$.

We examine the phase factors. The argument of the cosine function is given in terms of cyclotron frequency. In terms of magnetic field it is,

$$\theta = 2\pi p \left( \frac{2\pi mc}{h\nu_\pm eB} - \frac{1}{2} \right)$$

(7)

where $h$ is the Planck’s constant. The samples in which these experiments are done are usually made of heterostructures. The wire part of the sample is often very small so that there are small spaces where field is quantized. In very small constrictions, the field is quantized as $B.A=\pi\phi_0$ with unit flux quantum, $\phi_0=hc/e$, so that $B$ can be eliminated. The periodicity in the Shubnikov-de Haas oscillations is then changed by the “flux quantization”. For $\nu_\pm=1$, $n_2=1$, $p=1$, we obtain the phase factor as,

$$\theta = 2\pi \left( \frac{2\pi m A}{h^2} - \frac{1}{2} \right)$$

(8)

so that instead of mass of the electron, $m/h^2$, can be measured. We have found[10] that the flux quantization substituted in the classical Hall effect leads to the ‘quantum Hall effect’. Similarly, the flux quantization appearing in the Shubnikov-de Haas oscillations will now be called “quantum Shubnikov-de Haas effect”. The exponential factor in (2) upon flux quantization becomes,

$$\exp(-\frac{p\pi mc}{\nu_\pm eB}) = \exp(-\frac{\pi n A}{\nu_\pm e B})$$

(9)

so that the charge cancels and only mass gets quantized. Similarly, introducing the flux quantization in the argument of the sinh factor we obtain,

$$h\nu_\pm\omega_\sigma = \frac{h\nu_\pm n_2\hbar}{mA}$$

(10)

so that the oscillating resistivity becomes,

$$\delta\rho_{xx}(\varepsilon) = \rho_0\Sigma_{\alpha\beta}\gamma_{\alpha\beta}c_\alpha c_\beta\exp\left(-\frac{\pi n A}{\nu_\pm e B}\frac{h^2}{2}\right)$$

(11)

with,
from which charge has cancelled so that it is clearly different from \((2)\). Whereas, the usual de Haas-van Alphen effect depends on the charge of the electron, in the “quantized de Haas-van Alphen effect”, the charge is eliminated. The many-body effects of the Coulomb interaction or that of the electron-phonon interaction, are usually treated by a transformation and a perturbation theory, so that they tend to be small corrections to the unperturbed value. The present calculations give effects in the zeroeth order which are much bigger than those obtained by perturbation techniques. The system is subject to many-body interactions which reduce the single-particle energy and hence increase the mass. The exactness of \(3/2\) shows that it is not due to a conventional many body theory. The factor of 1.5 in \(m\) is too large to come from many-body perturbation theory.

**EXPERIMENTAL MEASUREMENTS**

Recently, the mass of the electron has been measured\([11]\) from the temperature dependence of the oscillation amplitude of the Shubnikov-de Haas oscillations in GaAs/AlGaAs. A lot of measurements give \(m \approx m_{\text{band}}\) in the range of densities from \(3.5 \times 10^{10} \text{ cm}^{-2}\) to about \(45 \times 10^{10} \text{ cm}^{-2}\). However, a few measurements at low densities are at variance. Apparently, the mass depends on density through, \(g^* \mu_B B_p = 4\pi \hbar^2 n / (2m^*)\) so that the factor measured is \(m^* g^*/n\). Therefore, it is important to consider the factors which multiply the cyclotron frequency due to the spin-charge effect. The SdH oscillations occur at \(\hbar \omega_c\) but the quantized SdH, oscillations occur at \(n \hbar^2 / mA\). Since \(\hbar \omega_c = n \hbar^2 / mA\), the two effects become difficult to resolve. In Fig.1, we show the mass of the electron compared with the band value in GaAs/AlGaAs as a function of carrier density. It is clearly seen that \(m\) as well as \(1.5m\), both are present in the data. The upper plot shows that the \(m^*\) varies from \(0.067 \times 0.91091 \times 10^{-27}\) g to \((3/2)\) of this value. Hence \(m^*\) as well as \(m^*/(2/3)\), both are present in the data. When the theory of Asgari et al\([12]\) is used, the data can be made to fit at low densities but then it does not fit at high densities. Similarly, other theories such as that of Kwon et al\([13]\) fit the data at high densities but not at low densities. The Fig.1 also shows a dashed line based on self-energy corrections of Kwon et al which does not fit the data. Therefore, it is reasonable to predict that factors like \(\nu_+\) which are due to spin-charge effect given in ref.1 are indeed present in the data of SdH effect. Accordingly, the presence of \(\nu_+ = 1\) and \(2/3\) are marked in the Fig.1.

**SPIN-CHARGE LOCKING**
Recently, Andeson[14] has suggested that the charge of the electron may be described by matrices, just as the angular momentum is. The dot product of spin

**Figure 1.** The mass of the electron relative to band value, as a function of the carrier density deduced from the Shubnikov-de Haas effect in GaAs/AlGaAs heterostructures. The factors 1 and 2/3 are found to arise from the spin-charge effect of ref.1. The experimental data is obtained from Tan et al[11]. The interpretation is ours. The dashed line is found by Kwon et al[13] on the basis of small self-energy corrections due to many-body perturbative interactions. The factors of 1 and 2/3 in the mass are found by us in the present work. The mass of the free electron is $m_e$ and the screening radius is deduced from the density, $n$, per unit area.
and charge matrices will then appear in the wave function of a superconductor. When spin is aligned along the charge, such as \( s_x \) parallel to \( e_x \), the arrangement is called *spin-charge locking*. In our work refs.1-2,15-18, similar phenomenon occurs with the fractional charges which are identified in the quantum Hall effect experiments because the Hall resistivity is strictly \( h/e^2 \). In our calculation, \((1/2)g_\pm\) becomes the charge of a particle because it multiplies the Bohr magneton. This idea gives two different values of the splitting factor for two different signs of the spin. Many experimentally measured properties of the 2-dimensional electron gas are correctly predicted\([15-18]\) by this type of spin-charge effect. As the spin value changes, the charge of the particle also changes. In Dirac equation, the sign of the energy determines the charge of the particle so that positive and negative energy solutions are associated with the electron and the positron. The sign of the energy gives the charge of the particle. In our theory, sign of the spin gives the charge of the particle, so that when we consider the Zeeman type of the Hamiltonian, the sign of the spin determines the sign of the energy and hence the sign of the charge. Therefore, our spin-charge effect is consistent with the Dirac equation. As the spin value changes, the charge of the particle also changes. For \( l = 0, s=-1/2 \), the charge becomes 1/3 and for \( s=+1/2 \) it becomes 2/3 such that the two fractions add up to unity. Let us find the dot product of spin and charge. In the present calculation, dimensionless charge is written in terms of \( l \) and \( s \) as,

\[
e^*/e= \frac{l+(1/2) \pm s}{2l+1}.
\]  

(13)

Multiplying it from right hand side by spin, \( s' \), we find,

\[
e^*s'= (2l+1)^{-1}(l+1/2 \pm s).s'= (2l+1)^{-1}(1.s'\frac{1}{2}(1).s'+s.s')
\]  

(14)

where (1) is a unit matrix. Therefore, the charge is written in terms of spin-orbit and spin-spin interactions. That is why, in a lot of calculations, there is no explicit need to write charge as a matrix except in the fractional quantum Hall effect.

**CONCLUSIONS**

We find that \( m/\nu_\pm \) occurs in place of \( m \) for the mass of the electron in the Shubnikov-de Haas (SdH) effect. In fact, many other fractions of the mass of the electron, given in ref.1 become allowed so that the electron really “falls apart”. The oscillations due to flux quantization allow the measurement of \( m/h^2 \). The flux quantization in the Shubnikov-de Haas effect leads to “quantized Shubnikov-de Haas effect”. Therefore, we observe consequences of the effect of flux quantization on the Shubnikov-de Haas oscillations. There are zeroes in
the resistivity at certain fields. There is a spin-charge effect so that the spin flip corresponds to a change in the charge.

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REFERENCES