Problem 7.2

Show that the ratio of the stimulated emission to spontaneous emission is equal to the number of photons per mode.

\[ \frac{dN_2}{dt} \bigg|_{\text{stimulated emission}} = -B_{21} N_2 \rho(v) = \frac{C^3}{8\pi\hbar v^3} A_{21} N_2 \rho(v) \], where \( B_{21} = \frac{C^3}{A_{21} 8\pi\hbar v^3} \)

\[ \frac{dN_2}{dt} \bigg|_{\text{spontaneous emission}} = A_{21} N_2. \]

\[ \frac{\text{stimulated rate}}{\text{spontaneous rate}} = \frac{\frac{C^3}{8\pi\hbar v^3} A_{21} N_2 \rho(v)}{A_{21} N_2} = \frac{\rho(v)}{8\pi\hbar v^3} = \frac{C^3}{8\pi\hbar v^3} \left( \frac{8\pi\hbar v^3 N_p}{C^3} \right) \]

\[ = N_p. \]

where, \( \rho(v) = \frac{8\pi\hbar v^3 N_p}{C^3} \)

\[ N_p = \frac{1}{e^{\hbar v/kT} - 1}. \]